# EFFECT OF THE MAGNETORESISTANCE ON LINER DESIGN

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***Abstract***

For beam particle dynamic stability, the inner wall of the liner tube (or beam tube) must meet the resistive wall requirement: conductance ($\sigma$) • thickness ($\delta$) $\geq 2 \times 10^5\ \Omega^{-1}$. Due to magnetoresistance, the $\sigma$ in collider operation will be smaller than that value without B fields. The magnitude of the change is dependent on: (1) the RRR of the material, (2) the operational temperature, and (3) the applied magnetic fields. This paper will briefly introduce the concept of magnetoresistance, and the effects of magnetoresistance on the choice of the RRR of the copper coating layer in both the case of a 4 K liner and of an 80 K liner. Calculations of the temperature increase of a liner tube during a magnet quench are presented. The Lorentz pressure is estimated for liners at different operational temperatures and with different copper thicknesses.

## 1.0 INTRODUCTION

The image current in the inner wall of a liner tube (or beam robe)is induced by the charged particle bunches that move along the center line of the liner (or beam) tube with a speed close to that of light. From the point of view of beam dynamic stability, the product of the conductance and thickness of the liner tube must meet the following requirements:

conductance ($\sigma$) • thickness ($\delta$) $\geq 2 \times 10^5\ \Omega^{-1}$. ID = 25 ~ 30 mm

conductance ($\sigma$) • thickness ($\delta$) $> 1\times 10^5\ \Omega^{-1}$. ID$\approx$40 mm

In the SSC Collider, the liner tube is under a strong magnetic field of 6.6 T. An extra resistance in a metal, a so called magnetoresistance, must be considered in the presence of a magnetic field. This magnetoresistance may be changed by a change in the direction of the magnetic field and/or its magnitude, depending on the Fermi surface topology. If there is a distribution of electron velocities, then it is clear that only those electrons of a certain "average" velocity will be undeflected. The remaining carriers, having velocities either larger or smaller than the "average" will be deflected and will traverse longer paths, thus increasing the resistance of the conductor. We have

$$\Delta\rho/\rho = aB^2 / (1 + \mu^2 B^2) \tag{1}$$

where $a = 0.38\mu^2 \times 10^{-16}$,

$\mu$ - Hall mobility, $cm^2/V\text{-}s$,

B - Oe.

For the reasons discussed and the fact that $\triangle\rho/\rho_o$ varies as $B^2$ in weak fields, magnetoresistance is a second-order effect, It tends to saturate in strong magnetic fields, unless there is a disturbance of the Hall field. Magnetoresistance is even in B. and it is related to the symmetric components of the resistivity tensor.

Also, we know the magnetoresistance will increase both with a reduction in temperature and with an increase in the material's RRR. The magnetotesistance can be varied by a factor of 0.5-10 at 4 K, and by a factor of 0.1-0.2 at 80 K. A good amount of experimental data is available at 4 K, but little is available at 80 K. For reference, we calculated from experimental data and assumptions the relative magnetorrsistance, $\triangle R/R_o$, as functions of the copper RRR of the liner tube at 4 K, 30 K, and 80 K under magnetic fields of 5 T and 6 T, as summarized

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in Figure 1. The detail considerations of the effects of the magnetoresistance on the liner design are discussed in the following sections.

(calculated by Q. S. Shu and K. Yu, 4/93 )

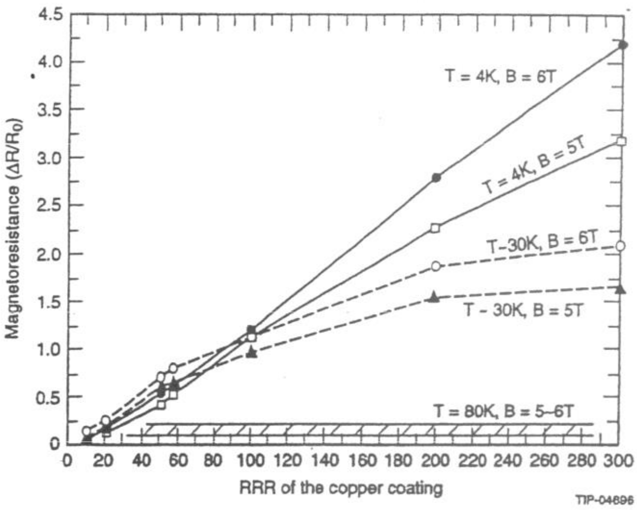


**Figure 1. Copper Magnetoresistance vs. H, T, and RRR.**

## 2.0 EFFECT OF MAGNETORESISTANCE ON CHOICE OF RRR AND COPPER THICKNESS OF THE LINER TUBE

As mentioned, if the liner (or beam) tube ID=25 ~ 30 mm,

$$\sigma(B,T) \cdot \delta \geq 2 \times 10^5\ \Omega^{-1}. \quad (2)$$

It is essential to use the real copper resistance values for the liner tube under magnetic fields to meet the resistive wall requirement

Definition,

$RRR = \rho(0,273) \cdot \sigma(0,4)$ (3)

$RR = \rho(0,273) / \rho(0,T)$ (4)

$\triangle R/R_0 = \rho(B,T) - \rho(0,T) / \rho(0,T)$ (5)

$\rho(0,273) = 1.545 \times 10^{-8}\ (\Omega.m)$

$\sigma(B,T) = \sigma(0,T) / [\triangle R/R_0 + 1].$ (6)

### 2.1 In the Case of a 4 K Liner(or Beam Tube)

Temperature, T=4K

Magnetic field, B=6.6T

Proposed copper coating layer thickness, $\delta$=0.1mm

1) If the effect of magnetoresistance in not taken into account in determining the minimum required RRR:

$\sigma(0,4) \geqslant 2.0 \times 10^9 (\Omega.m)^{-1}$

$RRR = 2.0 \times 10^9 (\Omega.m)^{-1} \cdot \rho(0,273) = 31$

2) If the effect of magnetoresistance is taken into account in determining the minimum required RRR:

$\sigma(6.6,4) \geqslant 2.0 \times 10^9 (\Omega.m)^{-1}$

$\sigma(B,T) \cdot [\triangle R/R_0 + 1] = \sigma(0,T)$

$\sigma(6.6,4) \cdot [0.6+1] = RRR / [\rho(0,273)]$

$RRR = 2.0 \times 10^9 (\Omega m)^{-1} \cdot \rho(0,273) \times 1.6 = 50$

To satisfy the resistive wall requirement, the RRR must be ⩾50, but not = 31.

3) Now,we would like to recommend the choices of copper RRR and coating thickness under the different resistive wall requirements in Figure 2.

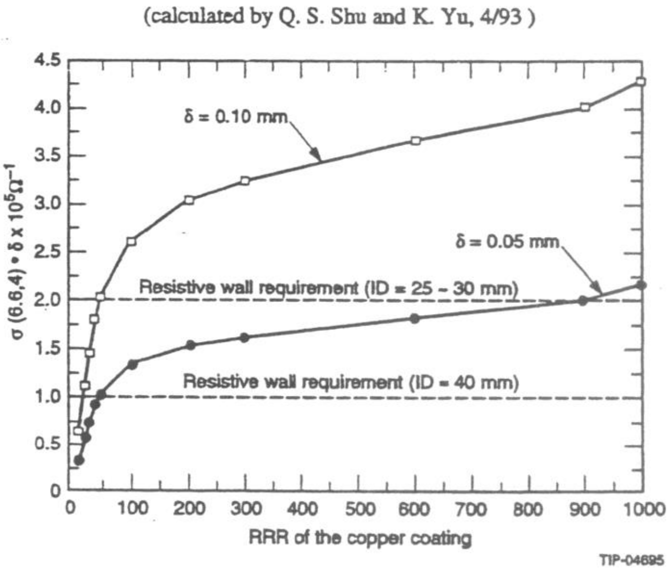


**Figure 2. RRR vs.s(B=6.6T,T=4K) • d(m) $\times 10^5$W$^{-1}$**

For example:

If $\sigma(B = 6.6T, T = 4K) \bullet \delta(m) \geq 2\times10^{-5}\ \Omega^{-1}$, $\delta$=0.1mm, RRR≥50

$\delta$=0.05mm, RRR≥900

If $\sigma(B = 6.6T, T = 4K) \bullet \delta(m) \geq 1\times10^{-5}\ \Omega^{-1}$, $\delta$=0.1mm, RRR⩾20

$\delta$=0.05mm, RRR⩾50

**2.2 In the Case of an 80 K Liner**

Temperature T=80K

Magnetic field B=6.6T

Possible copper coating layer thickness $\delta$=0.5mm, 0.55mm, 0.60mm

From graphs in Appendices B and C,when RRR=20,

$\sigma(0,80) \approx 3.45\times10^{8}(\Omega.m)^{-1}$

If $\triangle R/R_0 \approx 0.1$, $\sigma(6.6,80) \approx 3.14\times10^{8}(\Omega.m)^{-1}$

If $\triangle R/R_0 \approx 0.2$, $\sigma(6.6,80) \approx 2.88\times10^{8}(\Omega.m)^{-1}$

The($\sigma \bullet \delta$)can be calculated with different RRR and $\triangle R/R_0$, as shown in Table 1.

1) If magnetoresistance is $\triangle R/R_0 \approx 0.10$ at T=80K, B=6.6T, to meet the resistive wall requirement, the thickness vs. the required minimum RRR is listed as follows:

$\delta$=0.50mm RRR≥54

$\delta$=0.55mm RRR≥50

$\delta$=0.60mm RRR⩾32

2) If magnetoresistance is $\triangle R/R_0 \approx 0.20$ at T=80K, B=6.6T, to meet the resistive wall requirement, the thickness vs.the required minimum RRR is listed as follows:

$\delta$=0.50mm RRR≥77

$\delta$=0.55mm RRR≥54

$\delta$=0.60mm RRR⩾50

**TABLE 1.THE VALUE OF s(B = 6.6T, T = 80K) • d(m).**

| RRR | $\triangle R/R_0$ | s(0,80)×$10^8$ $(M.\Omega)^{-1}$ | s(6.6,80)× $10^8(M.\Omega)^{-1}$ | s(6.6,80) • d×$10^5\Omega^{-1}$ | | |
|---|---|---|---|---|---|---|
| | | | | d=0.50mm | d=0.55mm | d=0.60mm |
| 20 | 0.1 | 3.45 | 3.14 | 1.57 | 1.73 | 1.88 |
| 50 | | 4 | 3.64 | 1.82 | 2.01 | 2.18 |
| 57 | | 4.6 | 4.18 | 2.09 | 2.3 | 2.51 |
| 100 | | 5 | 4.55 | 2.28 | 2.5 | 2.73 |
| 20 | 0.2 | 3.45 | 2.88 | 1.44 | 1.58 | 1.73 |
| 50 | | 4 | 3.33 | 1.67 | 1.83 | 2 |
| 57 | | 4.6 | 3.83 | 1.92 | 2.11 | 2.3 |
| 100 | | 5 | 4.17 | 2.08 | 2.29 | 2.5 |

## 3.0 TEMPERATURE INCREASE OF THE COPPER COATING OF A LINER DURING MAGNET QUENCH

The quench performance of an ID=50 mm dipole magnet, DCA320,was calculated from experimental data.When discharge time is t = 0.18s, the B(t) • [dB(t)/dt] has reached its maximum and the Lorentz force is $F_{max}$.Att = 0.18s, we have:

B(t)=6.01 T

dB(t)/dt=24.52 T/s

B(t) • $[dB(t)/dt]/_{max}$=147.26$T^2$/S.

As shown in Figure 3(a),(b),(c).

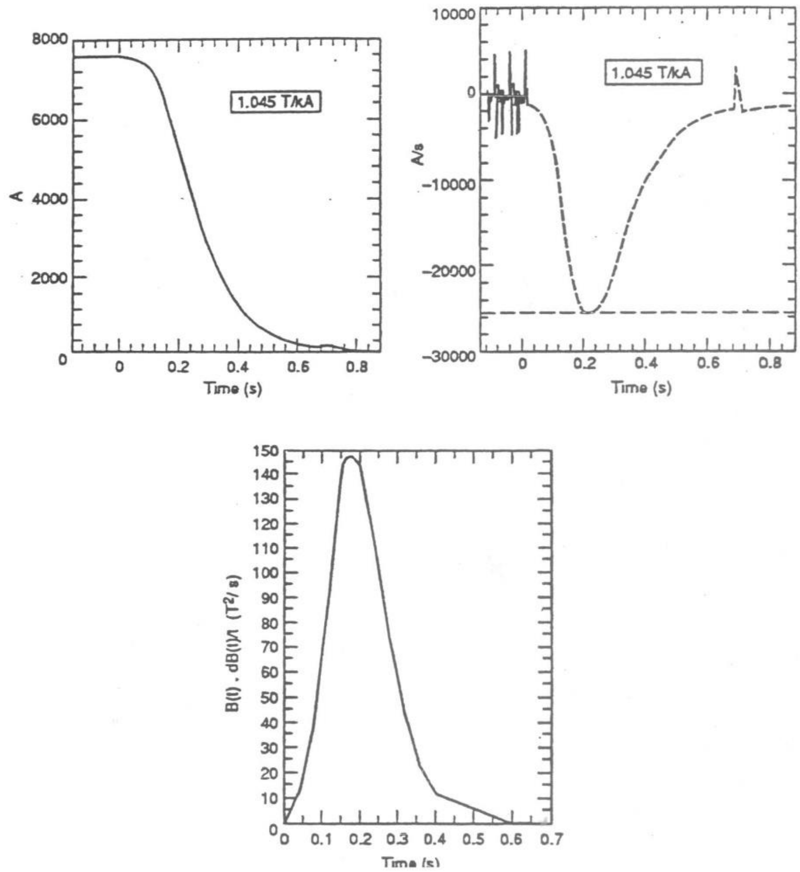


**Figure 3. The Above Experimental Quench Data Were Taken from the SSC CDM DCA320.(a) Current decay after quench.(b)Current decay rate as function of time.(c)B(t) • [dB(t)/dt] as function of time.**

### 3.1 In the Case of a 4 K Liner

The heat change between the copper coating and the stainless steel tube is ignored.

Heat capacity of copper at 40 K $C_p = 60$(J/kg.K)

Copper density $\zeta = 9.02\times10^3$ (kg/m$^3$)

Eddy current density $J(t) = \sigma(B,T)\text{-}\ \{[dB(t)]/dt\}\cdot r'$ (7)

Proposed copper coating thickness $\delta = 0.1$mm

ID=25.3mm ($r_1 = 1.265\times10^{-2}$ m)

OD = 25.5 mm ($r_2 = 1.275\times10^{-2}$ m)

Mean radius of copper coating layer $r' = (r_1 + r_2)/2 = 1.27 \times 10^{-2}$ m

Copper mass M

RRR=57

Electrical conductivity $\sigma(6,40) = 14.7\times10^8\ (\Omega.\text{m})^{-1}$

At the midplane $\sin\varphi = 90°$

On account of $C_pMdT = I^2Rdt$ (8)

$= [(J\cdot S)^2 \cdot \rho \cdot L/S]$at

$= (J^2SL/\sigma)dt$

$= (J^2V/\sigma)dt$

$C_pdT = (J^2/\sigma) \cdot (V/M)dt$ (9)

$= (J^2/\sigma) \cdot (1/\zeta)dt$

$\therefore$ $dT/dt = J^2/(\sigma \cdot \zeta \cdot Cp)$ (10)

= 263 K/s.

When the Lorentz force reaches maximum, discharge time is dt =0.18 s.

$\therefore$ $dT \approx 47$ K

Therefore, the temperature of the liner tube is T≈4 K + 47 K =51 K during the quench (at dt = 0.18 s).

**Note:** The approximation method of successive substitution is utilized in choosing the "heat capacity of copper at 40 K" and the "electrical conductivity σ(6, 40)" for the above calculation of temperature increase.

### 3.2 In the Case of an 80 K Liner

The heat change between the copper coating and the stainless steel tube is ignored.

Heat capacity of copper at 80 K $C_p = 200$ (Jkg.K)

copper density $\zeta = 9.02\times10^3$ (kg/m$^3$)

Eddy current density $J(t) = \sigma(B,T)\cdot\{[dB(t)]/dt\}\cdot r'$

Proposed copper coating thickness $\delta = 0.5$ mm

ID = 25.3 mm ($r_1 = 1.265\times10^{-2}$ m)

OD = 26.3 mm ($r_2 = 1.315\times10^{-2}$ m)

Mean radius of copper coating layer $r' = (r_1 + r_2)/2 = 1.29\times10^{-2}$m

Copper mass M

RRR= 57

Electrical conductivity (if $\triangle R/P_0 \approx 0.10$ )

$\sigma(6,80)=4.2\times10^8\ (\Omega.\text{m})^{-1}$

At the midplane $\text{sim}\,\varphi = 90°$

use

$C_pdT=J^2/(\sigma \cdot \zeta)dt$

$dT/dt= J^2/(\sigma \cdot \zeta \cdot C_p)$

= 27 K/s.

When the Lorentz force reaches maximum, discharge time is dt = 0.18 s.

$\therefore \quad dT \approx 5K.$

Therefore, the teperature of the linertube is T≈80K+ 5 K=85 K during the quench (at dt = 0.18 s).

## 4.0 EFFECT OF T AND δ ON LORENTZ PRESSURE

1) Lorentz pressure $P_{max}$:

The differential Lorentz force

$$dF = I \times B(t) = I \cdot B(t) \sin\Omega \qquad (11)$$

due to $F = \iint 1/[1 + \pi(r/1) \cdot (\varphi/90)]\, \sigma(B,T) \cdot B(t) \cdot [dB(t)]/dt \cdot r^2 \cdot dr \cdot \text{sim}\varphi \cdot d\Phi \qquad (12)$

$F_{max} = \sigma(B,T) \cdot \{B(t) \cdot [dB(t)/dt]\}|_{max} \cdot \iint 1/[1 + \pi(r/1) \cdot (\varphi/90)] \cdot r^2 \cdot dr \cdot \sin\varphi \cdot d\Phi \quad (13)$

$$P_{max} = F_{max}/(2 \times r' \times \Phi) \qquad (14)$$

If $\delta << r$, $r << 1$ and $\varphi = 90^\circ$

$$\therefore \quad F_{max} = \sigma(B,T) \cdot \{B(t) \cdot [dB(t)/dt]\}|_{max} \cdot \iint r^2 \cdot dr \cdot d\Phi \qquad (15)$$

we have $P_{max} = \sigma(B,T)\{B \cdot [dB/dT]\}|_{max} \cdot r' \cdot \delta \qquad (16)$

The detailed deduction is presented in the Appendix A.

### 4.1 In the Case of a 4 K Liner

If we choose: RRR = 57

ID = 25.3 mm ($r_1 = 1.265 \times 10^{-2}$ m)

Copper coating thickness δ= 0.10 mm, 0.5 mm

Electrical conductivity $\sigma(6,4) \approx 18 \times 10^8\ (\Omega.m)^{-1}$

$\sigma(6,51) \approx 10.8 \times 10^8\ (\Omega.m)^{-1}$

$P_{max}$ = 49 Psi, when δ = 0.10 mm, r' = $1.27 \times 10^{-2}$ m.

If the temperature increase of the copper coating layer is taken into account during magnet quench, $P_{max} \approx 30$ Psi.

$P_{max} \approx 248$ Psi, when δ = 0.50 mm, r' = $1.29 \times 10^{-2}$ m.

If the temperature increase of the copper coating layer is taken into account during magnet quench, $P_{max} \approx 149$ Psi.

### 4.2 In the Case of an 80 K liner

If we choose: RRR = 57

ID = 25.3 mm ($r_1 = 1.265 \times 10^{-2}$ m)

Copper coating thickness δ= 0.50 mm, 0.55 mm, 0.60 mm

Suggested magnetoresistance $\triangle R/R_0 \approx 0.10$

Electrical conductivity $\sigma(6,80) \approx 4.2 \times 10^8 (\Omega.m)^{-1}$

$\sigma(6,85) \approx 3.3 \times 10^8 (\Omega.m)^{-1}$

$P_{max} \approx 57$ Psi, when δ = 0.50 mm, r' = $1.29 \times 10^{-2}$ m.

If the temperature increase of the copper coating layer is taken into account during magnet quench,

$P_{max} \approx 46$ Psi.

$P_{max} \approx 64$ Psi, when δ = 0.55 mm, r' = $1.2925 \times 10^{-2}$ m.

If the temperature increase of the copper coating layer is taken into account during magnet quench,

$P_{max} \approx 50$ Psi.

$P_{max} \approx 70$ Psi, when δ = 0.60 mm, r' = $1.295 \times 10^{-2}$ m.

If the temperature increase of the copper coating layer is taken into account during magnet quench,

$P_{max} \approx 55$ Psi.

## REFERENCES

[1] Q.S. Shu, "Report on the ASSTII Liner Status," SSCL-N-805, Nov. 1992.

## APPENDIX A

A Lorentz pressure, $P_{max}$, is estimated on the liner tube during a magnet quench for the SSC dipole. The Lorentz force formula was deduced as below.

$$dF = I \times B(t) = I \cdot B(t) \sin\Omega \qquad (A.1)$$

because, $\Omega = \pi/2$

$$I = j(t) \cdot ds = j(t) r \cdot dr \cdot d\Phi . \qquad (A.2)$$

To put Eq. (A.2) into Eq. (A.1), the force per unit length is given by:

$$dF = j(t) \cdot B(t) \cdot r \cdot dr \cdot d\Phi , \qquad (A.3)$$

and due to

$j(t) = \sigma(B,T)E(t)$

$I(t) = \varepsilon(t)/R$

$R = \rho L/S = L/[\sigma(B,T) \cdot S]$

$I(t) = \varepsilon(t)/\{ L/[\sigma(B,T) \cdot S] \}$

$I(t)/S = \varepsilon(t)\ \sigma(B,T)/L$

$\therefore \quad j(t) = [\sigma(B,T) \cdot \varepsilon(t)]/L \qquad (A.4)$

therefore, $L = 2l + 2\pi r \cdot 2\varphi/180$ (m)

$= 2[l + \pi r\varphi/90]. \qquad (A.5)$

To put Eq. (A.5) into Eq. (A.4),

$j(t) = 1/[1 + \pi r\varphi/90] \cdot [\sigma(B,T) \cdot \varepsilon(t)]/2, \qquad (A.6)$

and due to $\varepsilon(t) = -d\Phi / dt = -S' [dB(t)/dt]$

$S' = l \cdot 2r \cdot \sin\varphi$

$\therefore \quad \varepsilon(t) = 1\{[dB(t)/dt\} \cdot 2r \cdot \sin\varphi. \qquad (A.7)$

To put Eq. (A.7) into Eq. (A.6), the current density is given by:

$j(t) = 1/[1 + \pi (r/l) \cdot \varphi/90] \cdot \sigma(B,T) \cdot \{ [dB(t)/dt\} \cdot r \cdot \sin\varphi. \qquad (A.8)$

To put Eq. (A.8) into Eq. (A.3),

$dF = 1/[1 + \pi(r/l) \cdot \varphi/90] \cdot \sigma(B,T) \cdot B(t) \cdot [dB(t)]/dt \cdot r^2 \cdot dr \cdot d\phi \cdot \mathrm{sim}\varphi$

$F = \iint 1/[1 + \pi(r/l) \cdot \varphi/90] \cdot \sigma(B,T) \cdot B(t) \cdot [dB(t)]/dt \cdot r^2 \cdot dr \cdot d\phi \cdot \mathrm{sim}\varphi$

If the Lorentz force reaches maximum $F_{max}$, $B(t) \cdot [dB(t)/dt]$ must reach maximum, $\varphi = 90^\circ$ also. For this reason,

$F_{max} = \sigma(B,T) \cdot \{B(t) \cdot [dB(t)/dt] \}|_{max} \cdot \iint 1/[1 + \pi r/l] \cdot r^2 \cdot dr \cdot d\phi \qquad (A.9)$

$P_{max} = F_{max}/(2 \times r' \times \phi)$.

If $\quad \delta << r$, and $r << 1$

we have $\quad P_{max} = \sigma(B,T) \cdot \{B\text{-}[dB/dT] \}|_{max} \cdot r' \cdot \delta. \qquad (A.10)$

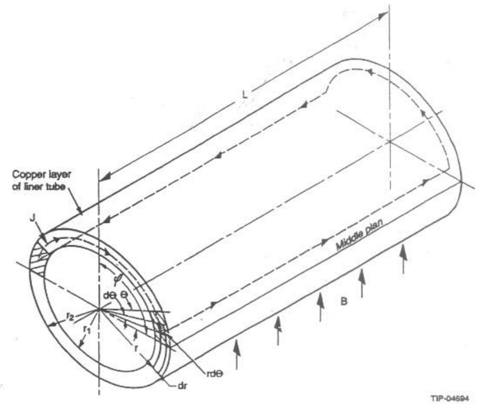


**Figure A-1. A Schematic of a Liner Copper Coating for Lorentz Pressure Calculation.**

## APPENDIXB

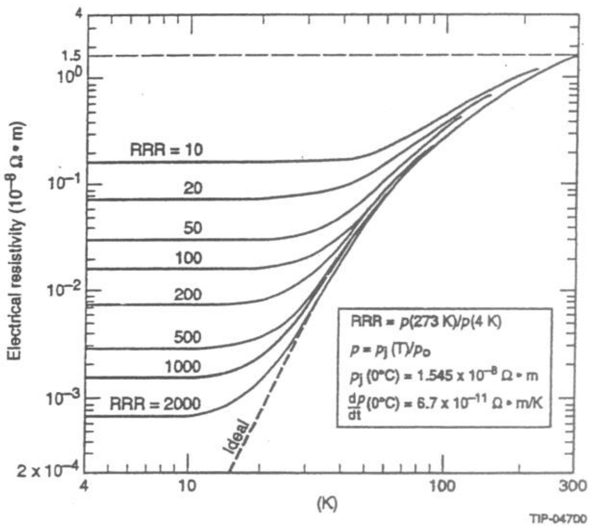


**Figure B-1. Graph of Copper Electrical Resistivity vs.Temperature at B=0(Taken from NIST).**

## APPENDIX B

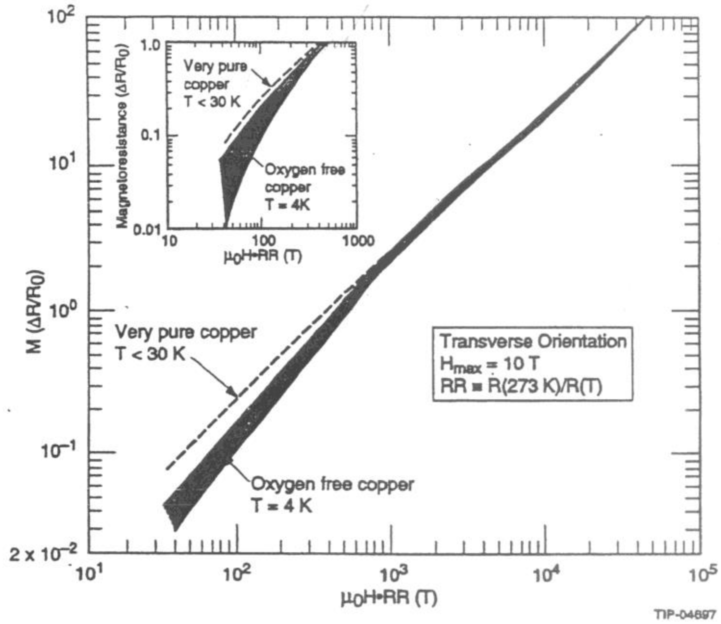


**Figure C-1. Graph of Fractional Change in Copper Electrical Resistance Under Transverse Magnetic Field (Taken from**

**NIST).**